

# Erasmus + projekt



Atena, Grčka

25.03.2015. – 31.03.2015.



AGENCIJA ZA  
MOBILNOST I  
PROGRAME EU

Naziv stručnog usavršavanja:

**LE MATH**

**Learning mathematics through  
new communication factors**



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Sudjelovalo je 10 polaznika iz 5 različitih zemalja, pa smo se prvo međusobno upoznali i predstavili zemlje iz kojih dolazimo.



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# Nakon toga smo se upoznali s temom tečaja:

## **MATHfactor**

teaching and learning  
mathematics through  
mathematics  
communication  
activities

## **MATHeatre**

teaching and learning  
mathematics through  
math theatre activities





Imali smo priliku prisustvovati natjecanju učenika u MATHeatru i u MATHfactoru, te smo kao pomoćni suci primijenili naučeno znanje i ocjenjivali učenike.



Pojedine izvedbe natjecanja, možete pogledati na sljedećim linkovima:

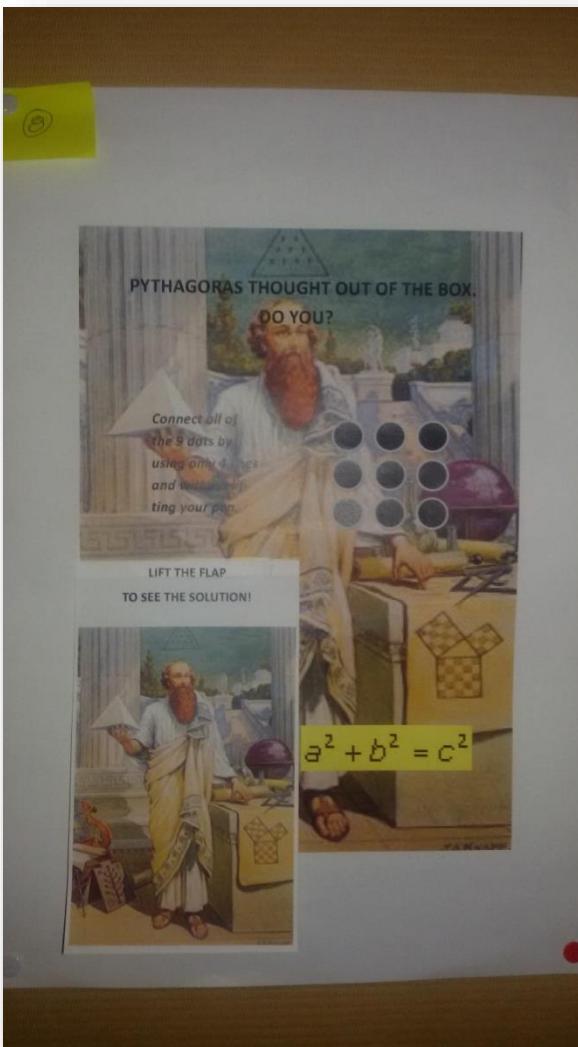
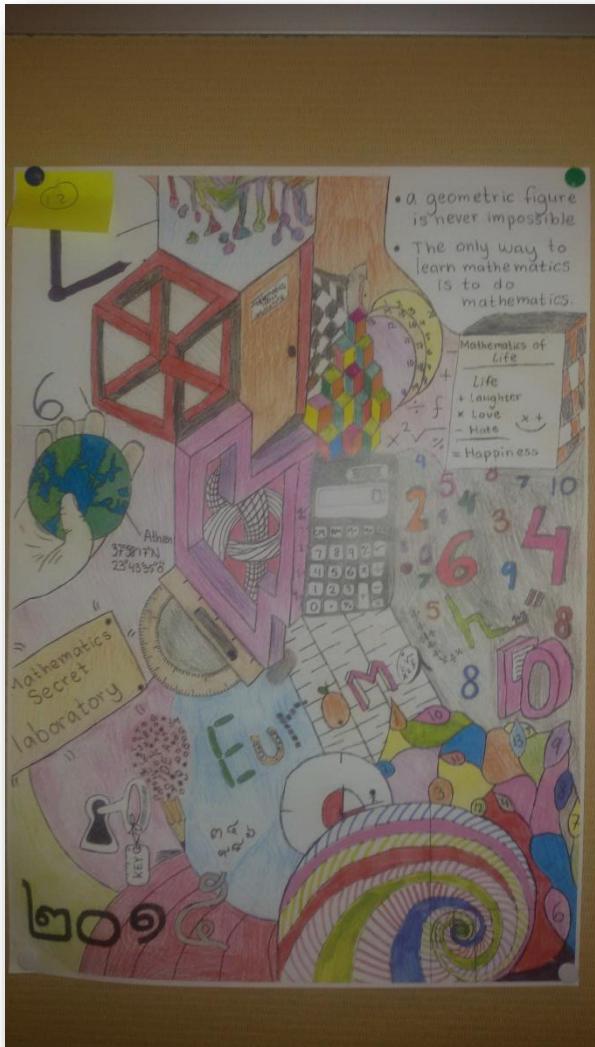
- <https://www.youtube.com/watch?v=qR1qtHDh8ik&list=PLpPvt2LgHCYcjZrGzCHMishvEadO6Kqpf>
- <https://www.youtube.com/watch?v=OM945oBQM5c&list=PLpPvt2LgHCYd1FyEv7D2DK55Txdn8DsNF>



# Imali smo priliku vidjeti i natjecanje u izradi matematičkih postera.



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**Configurations of lines and circles**  
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**EUROMATH**

**ATHENS 2015**

**Generalization of the Sylvester-Gallai Theorem**

Two points in a plane determine exactly one line. Similarly, three non-collinear points determine a single circle. Five, general enough points determine a conic, and more generally,  $n$  points in a plane determine a configuration of  $\binom{n}{2}$  lines. This has led to a theater play I wrote in 2013 [1], an article in 2014 [2] and interest in configurations of arbitrary lines. This is a continuation of that story...

**My path to maths**

see different ways and motivations to do mathematics. Nine originates in a computer game, nine trees in a legend. The tenth of the pane is a triangle with clearly marked diagonals. I started to draw configurations of lines and circles. This has led to a theater play I wrote in 2013 [1], an article in 2014 [2] and interest in configurations of arbitrary lines. This is a continuation of that story...

**Theorem (Sylvester-Gallai Theorem for points)**  
Let  $P$  be a finite set of points in the euclidean plane. Then:

- either all points are contained in a conic (they might be collinear);- or there exists a line passing through exactly 2 points from the set  $P$  and this line is unique for these particular 2 points;

This is so natural to ask if a similar property holds for circles. My main result is the following theorem showing that this is indeed the case.

**Theorem (Sylvester-Gallai Theorem for circles)**  
Let  $P$  be a finite set of circles in the euclidean plane. Then:

- either all points are collinear;
- or they are contained in a single circle;
- or there exists a circle passing through exactly 2 points from the set  $P$  and this circle is unique with the circles with the indices  $i$  and the radius  $r_i$ .

The main tool used in the proof of the above theorem is the inversion.

**The inversion**

The inversion in the plane is a transformation of the plane which can be viewed as a reflection along a circle (in contrast to the usual euclidean reflection along a line). So let  $(O, r)$  be a circle with center  $O$  and radius  $r$ .  
**Definition**  
Let  $P$  be a point in the plane distinct from the point  $O$ . Then, the  $P'$  of  $P$  under the inversion with respect to the circle  $(O, r)$  is such a point that

- The points  $O, P, P'$  are collinear;
- The point  $P'$  lies on the same side of the point  $O$  as the point  $P$ ;
- $|OP'|^2 = r^2$ .

**Properties of the inversion**

The inversion is a self-inverse transformation of the plane. The point  $O$  has no image.

**Images of lines and circles under inversion**

**Configuration of 9 trees with 10 lines containing 3 trees**

**Self-prepared graphics with Geogebra**

**References**

[1] Adam Czaplański, Małgorzata Dąbrowska, Lucja Fendt, James Gosselink, Magdalena Łapko, Barbara Matkowska, Grzegorz Mirek, Małgorzata Nowak, Justyna Słupińska, Małgorzata Tarczynska, *On the Sylvester-Gallai theorem for circles*, arXiv:1411.2448  
[2] Tobiasz Szemberg, *A mysterious man in a suit of scripte fuit offensio*, editor: Gábor Márki, ISBN 978-8363-3299-1-1, pp. 41–46  
[3] Tobiasz Szemberg, *Postav pravcević poslagajući u veličastog wojdu* (in Polish), available online

U slobodno vrijeme smo razgledavali  
Atenu i sprijateljili se ☺



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Više informacija o LE-MATH projektu možete pogledati na linku:

<http://www.le-math.eu/index.php?id=14>



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